Tissue Identification in Ultrasound Images using Rayleigh Local Parameter Estimation

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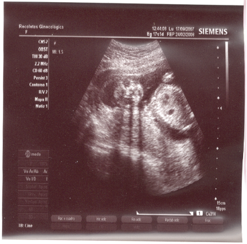
Speckle

- Speckle is a kind of granular noise,
- Found in many types of coherent imaging systems: SAR, laser illuminated or ultrasound.

Received signal model

$$Z = Xe^{j\phi} = \sum_{n=1}^{N} x_i e^{j\phi_i}$$

Speckle and Ultrasound



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Statistical Models for Speckle

Statistical models

1. Rayleigh: phases are random and independent of the amplitudes, number of scatters large, no periodicity in disposition of the scatters.

$$p(X) = \frac{X}{\sigma^2}\, e^{-\frac{X^2}{2\sigma^2}}\, u(X)$$

2. Rician distribution (specular component Z_s added).
3. K distribution
4. Homodyned K distribution.
5. Others: Nakagami model, Rician inverse Gaussian and Nakagami inverse Gaussian.

–Logarithmic compression no considered–

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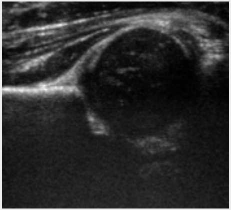
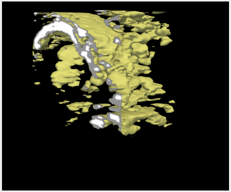
Aim of this work





Assumption

- Rayleigh distribution of the speckle.
- Uniform value of σ for each tissue; different values for different tissues.

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- Estimation of σ in each tissue.
- Classification of pixels according to $\hat{\sigma}$.

Tools

- Local Statistics

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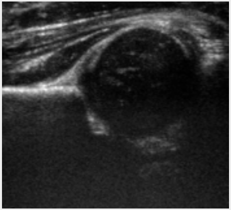

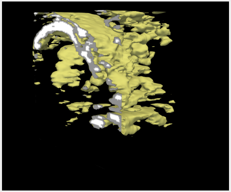

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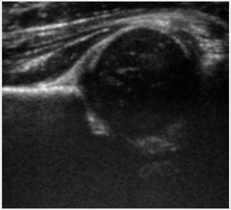

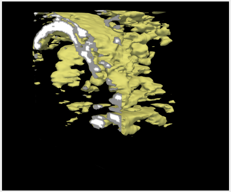

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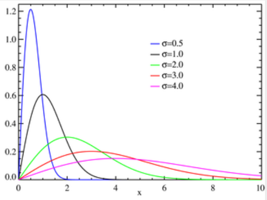
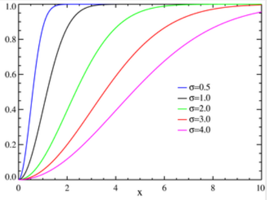
Local Statistics of the Rayleigh distribution

The Rayleigh distribution





PDF

$$p(x|\sigma) = \frac{x}{\sigma^2}\, e^{-\frac{x^2}{2\sigma^2}}\, u(x)$$

Moments and parameters

Mean: $E\{x\} = \sigma\sqrt{\frac{\pi}{2}}$

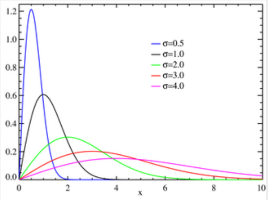
Median: $\text{median}\{x\} = \sigma\sqrt{\log(4)}$

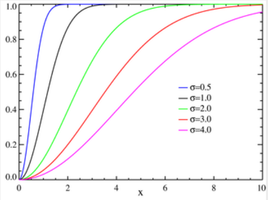
Mode: $\text{mode}\{x\} = \sigma$

Variance: $\text{Var}\{x\} = \sigma^2\frac{4-\pi}{2}$

Second order moment: $E\{x^2\} = 2\sigma^2$

The Rayleigh distribution





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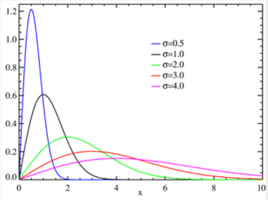

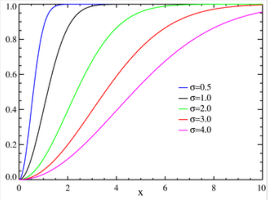

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Parameter estimation

Being $R_i(\sigma^2)$, $i = \{1, \cdots, N\}$ a set of random variables with Rayleigh distribution

Maximum Likelihood (ML) estimator

$$\widehat{\sigma^2}_{ML} = \frac{1}{2N} \sum_{i=1}^{N} R_i^2 = \frac{1}{2} \langle R_i^2 \rangle$$

Unbiased estimator

$$\widehat{\sigma}_c = \sqrt{\frac{2}{\pi}} \frac{1}{N} \sum_{i=1}^{N} R_i = \sqrt{\frac{2}{\pi}} \langle R_i \rangle$$

Parameter estimation (2)

ML estimator distribution

- Mean: $E\{\widehat{\sigma^2}_{ML}\} = \alpha\beta = 2\sigma^2$
- Mode: $\text{mode}\{\widehat{\sigma^2}_{ML}\} = (\alpha-1)\beta = \frac{N-1}{N}2\sigma^2$
- Variance: $\text{Var}\{\widehat{\sigma^2}_{ML}\} = \alpha\beta^2 = \frac{4\sigma^4}{N}$

Unbiased estimator distribution

- Mean: $E\{\widehat{\sigma}_c\} = \sigma\sqrt{\frac{\pi}{2}}$
- Mode: $\text{mode}\{\widehat{\sigma}_c\} \approx \sigma_n\sqrt{\frac{2(2N-1)N}{e}} \approx \sigma_n N\sqrt{\frac{\pi}{2}}$
- Variance: $\text{Var}\{\widehat{\sigma}_c\} = \frac{1}{N}\text{Var}\{R(\sigma)\} = \frac{1}{N}\sigma^2\frac{4-\pi}{2}$

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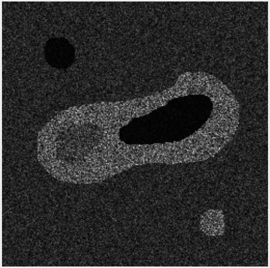
If $N >>$ mean and the mode are approximately equal.

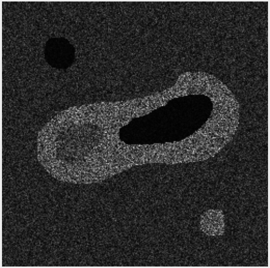
Tissue Identification in Ultrasound Images

Algorithm: about the image



- Assuming a Rayleigh model for the speckle
- Image model: $\sigma_{ij}^2 = f_{ij}^2 \sigma_n^2$
- Value of σ related to the characteristics of the biological tissue.
- Assumption: each scanned tissue uniform response: similar σ_{ij} for each tissue, and different for different tissues.

Local estimation of sigma allows the identification of different regions belonging to different types of tissues.



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Algorithm: overview

1. Original image estimation using local estimation and assuming Rayleigh distribution.
2. Classifying the pixels in tissues
    1. Defining the number of classes (tissues).
    2. Simple classification algorithm: K-means.

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Algorithm: image estimators

Mean

$$I_{\hat{\sigma}_{ij}} = \sqrt{\frac{2}{\pi}} \frac{1}{|\eta_{i,j}|} \sum_{p \in \eta_{i,j}} I_p$$

Squared Mean (ML)

$$I_{\hat{\sigma}_{ij}} = \sqrt{\frac{1}{2|\eta_{i,j}|} \sum_{p \in \eta_{i,j}} I_p^2}$$

Median

$$I_{\hat{\sigma}_{ij}} = \sqrt{\frac{1}{\log(4)}} \operatorname*{median}_{p \in \eta_{i,j}} \{ I_p \}$$

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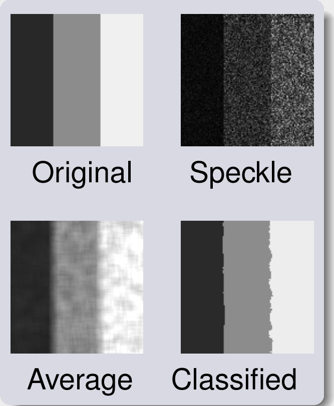
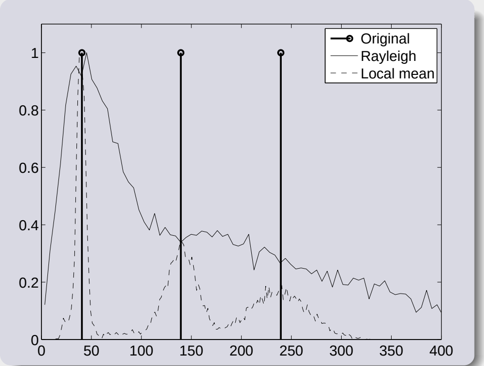
Algorithm: example

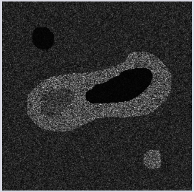




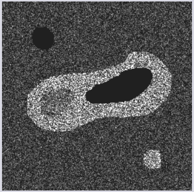
Experiments

Synthetic Experiments

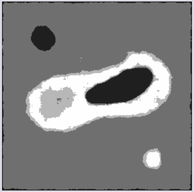
Original Image



With Speckle (Rayleigh)

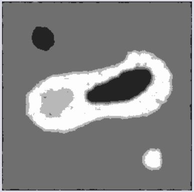


Clustering over speckle



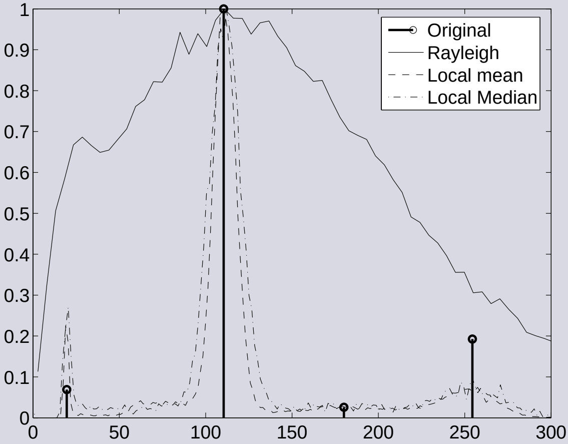
Clustering over median

Clustering over mean (9)

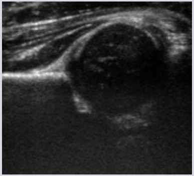


Clustering over mean (5)

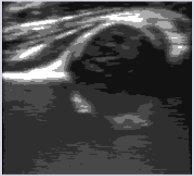
Synthetic Experiments



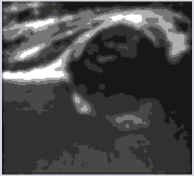
Experiments with Ultrasound Images



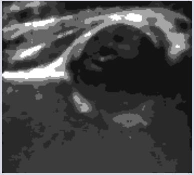
Original



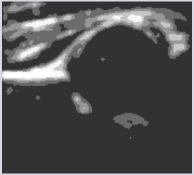
3D mean



2D mean



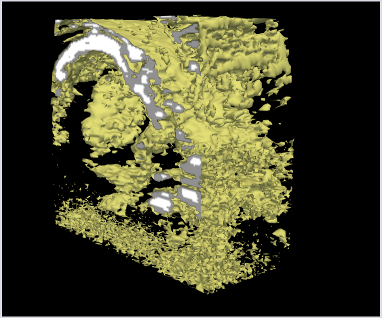
2D median



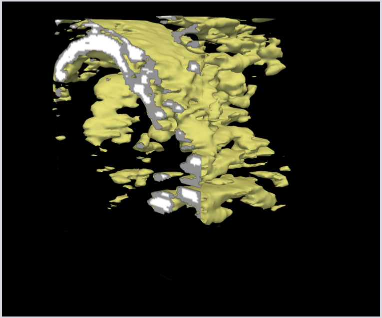
2D squared mean

Experiments with Ultrasound Images

Original



Processed



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- Make any subsequent segmentation easier.
- Clustering done with K-means, (though more complex algorithms may be used).

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- Using the information for more complex segmentation (Snakes?)
- Other distributions.
- Anisotropic estimation of the parameters.
- Logarithmic compression of the data.

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