Effect of Sampling on the Estimation of the Apparent Coefficient of Diffusion in MRI

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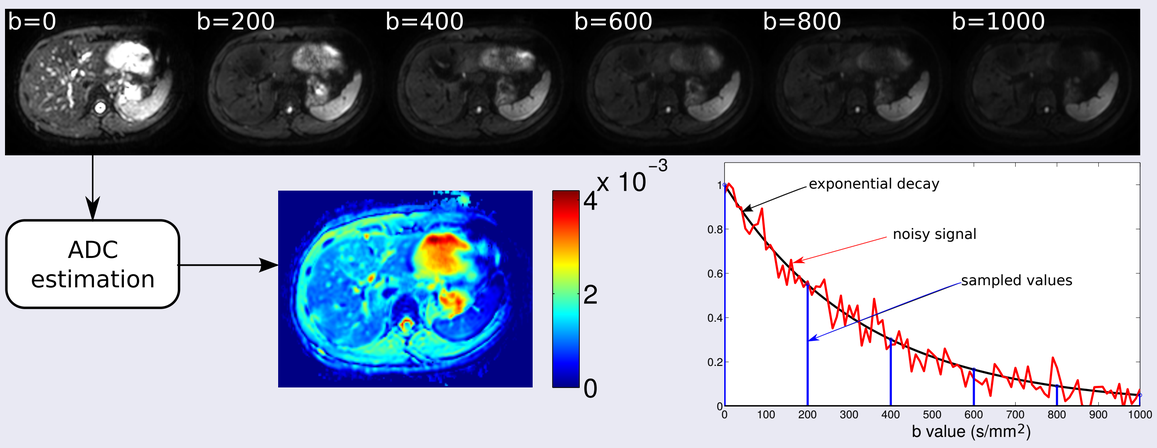
The Apparent Diffusion Coefficient (ADC) is a quantitative measure derived from Diffusion Weighted MRI that is able to asses the amount of water diffusion within living tissues. It is widely used in the clinical routine for diagnosis, characterization of different diseases and to evaluate response to therapy. For ADC estimation, the diffusion signal needs to be sampled using a small number of values, presenting distortions due to the aliasing and windowing effect. In this work, we theoretically study these effects and propose some new robust estimators for ADC based on the Fourier Transform of the signal.

Apparent Diffusion Coefficient (ADC)



- Quantitative Biomarker: accurate, precise, robust and reproducible.
- ADC estimated from small number of samples affected by multiple artifacts: motion-related errors, image distortions and noise-related effects.
- Others sources of error: number and positions of the samples available ⇒ aliasing, sampling and windowing.

Sampling of the Diffusion Signal

Diffusion signal:

$$S(b) = S_0 \cdot e^{-b\cdot \mathrm{ADC}}, \tag{1}$$

S_0: signal intensity at $b=0$; $S(b)$ signal intensity at b; ADC: apparent diffusion coefficient. Normalized by the baseline:

$$x(b) = e^{-b\cdot \mathrm{ADC}}\, u(b), \tag{2}$$

with FT calculated over the variable b:

$$X(\omega) = \frac{1}{\mathrm{ADC}+j\omega}, \quad X(0) = \int_{-\infty}^{\infty} x(b)db = \frac{1}{\mathrm{ADC}}. \tag{3}$$

However, in order to estimate the ADC a sampled version of $x(b)$ must be considered. As a result, the FT of the sampling signal will differ.

Uniform Sampling: $x(b)$ is uniformly sampled for equally spaced values of variable b, Δb, obtaining the discrete signal $x[n] = x(n\cdot\Delta b)$. The continuous sampled signal, $x_p(b)$ becomes:

$$x_p(b) = x(b)\cdot\sum_n \delta(b-n\Delta b),$$

with FT:

$$X_p(\omega) = \frac{1}{\Delta b}\frac{\pi}{\omega_s}\coth\left((\mathrm{ADC}+j\omega)\frac{\pi}{\omega_s}\right). \tag{4}$$

The FT in the origin will therefore be

$$X_p(0) = \frac{1}{\Delta b}\frac{\pi}{\omega_s}\coth\left(\mathrm{ADC}\frac{\pi}{\omega_s}\right). \tag{5}$$

where $\omega_s = \frac{2\pi}{\Delta b}$ is the sampling frequency.

Effect of Windowing: The original signal $x(b)$ is limited in b:

$$x(b) = \begin{cases} \exp(-b\cdot\mathrm{ADC}) & 0 \le b \le B_M \\ 0 & b<0,\ b>B_M \end{cases}$$

The FT of the continuos signal now becomes:

$$X(\omega) = \frac{1}{\mathrm{ADC}+j\omega}\left(1-e^{-B_M(\mathrm{ADC}+j\omega)}\right), \quad X(0) = \frac{1}{\mathrm{ADC}}\left(1-e^{-B_M\cdot\mathrm{ADC}}\right). \tag{6}$$

Windowing + sampling:

$$X_p(\omega) = \frac{1}{\Delta b}\left(1-e^{-B_M(\mathrm{ADC}+j\omega)}\right)\times\frac{\pi}{\omega_s}\coth\left((\mathrm{ADC}+j\omega)\frac{\pi}{\omega_s}\right). \tag{7}$$

$$X_p(0) = \frac{\pi}{\Delta b\cdot\omega_s}\left(1-e^{-B_M\mathrm{ADC}}\right)\coth\left(\mathrm{ADC}\frac{\pi}{\omega_s}\right). \tag{8}$$

FT of the discrete signal: Alternatively, we analyze the discrete signal from the samples:

$$x[n] = \exp(-n\Delta b\cdot\mathrm{ADC}), \quad n=0,\cdots,N-1.$$

The FT in the origin becomes:

$$X(0) = \sum_{n=0}^{N-1} x[n] = \sum_{n=0}^{N-1} e^{-n\Delta b\cdot\mathrm{ADC}} = \frac{1-e^{-N\Delta b\cdot\mathrm{ADC}}}{1-e^{-\Delta b\cdot\mathrm{ADC}}} = \frac{1-e^{-B_M\cdot\mathrm{ADC}}}{1-e^{-\Delta b\cdot\mathrm{ADC}}}. \tag{9}$$

Application to ADC estimation

- Direct application: derivation of new estimators for the ADC. Modifications and corrections of existing estimation methods are also possible.
- Estimators based on the center of the Fourier space: the point with higher SNR and less affected by aliasing. ⇒ robustness properties.

Aliasing model:

$$\widehat{\mathrm{ADC}} = \frac{1}{\Delta b}\log\left(\frac{S_b+\Delta b/2}{S_b-\Delta b/2}\right) \tag{10}$$

Aliasing and windowing model:

$$\widehat{\mathrm{ADC}} = \arg\min_y \left[S_b - \left(1-e^{-B_M y}\right)\frac{\pi}{\omega_s}\coth\left(y\frac{\pi}{\omega_s}\right)\right]^2 \tag{11}$$

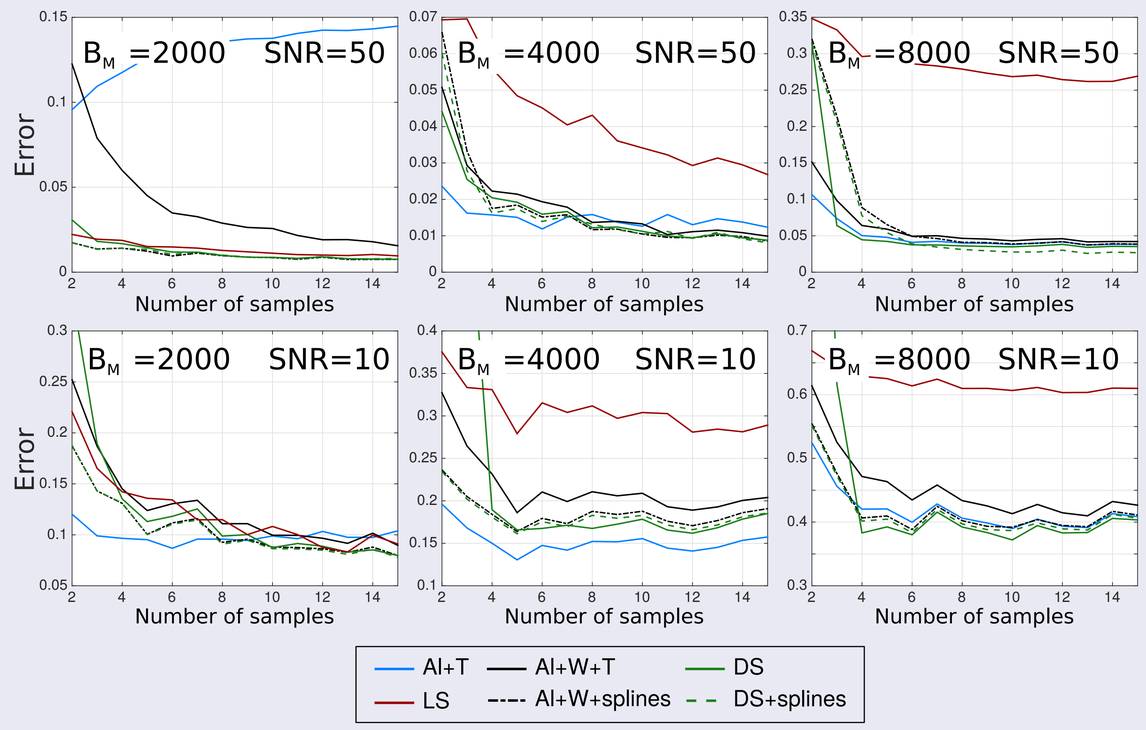
Discrete model:

$$\widehat{\mathrm{ADC}} = \arg\min_y \left[\sum_{n=0}^{N-1} x[n] - \frac{1-e^{-B_M\cdot y}}{1-e^{-\Delta b\cdot y}}\right]^2 \tag{12}$$

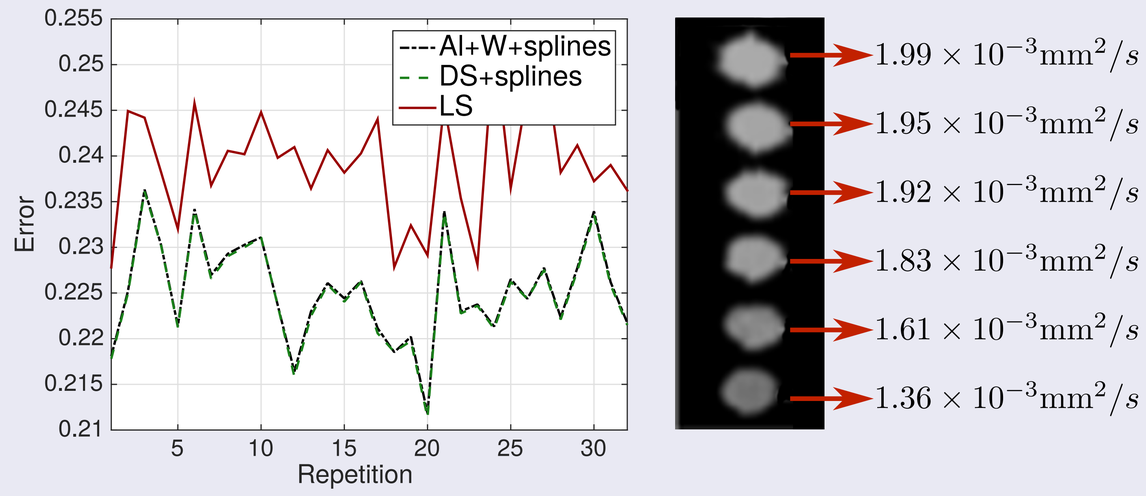
S_b will denote the area of the signal $S_b = \int_{-\infty}^{\infty} x_R(b)db$ and $x_R(b)$ as a low pass filtered interpolation of $x_p(t)$.

Results

Synthetic experiment: 1D signal, ADC$= 10^{-3}$mm^2/s corrupted with Rician noise, SNR=[10–50], $B_M = [2,4,8]\cdot 10^3$, variable number of samples (2 to 15), uniform sampling, average of 1000 experiments. [LS: least squares; AI: aliasing considered; W: windowing considered; DS: discrete summation; T: trapezoid functions].



Real phantom: 6 vials with agar-based oil-water emulsions with decreasing ADC (measured experimentally) in the range 2-1.3$\times 10^{-3}$ mm^2/s. Data acquired in a 1.5T MRI Scanner (Signa Hdxt, GE Healthcare) with a single-channel head coil and SS-EPI. Othe values: $b: 0,250,500,750,1000$ s/mm^2, slice thickness: 4mm, axial orientation. 32 repetitions of the same slice were acquired.



- Proposed estimators show improved robustness with respect to LS.
- Proper modeling of the sampled diffusion signal can help increase the accuracy of ADC estimation, and can be easily applied to modify or fine tune other related methods.

Acknowledgements

The authors acknowledge MICIN (Spain) for grants TEC2013-44194-P and TEC2014-57428. Authors thank Dr. Diego Hernando (Departments of Radiology and Medical Physics, University of Wisconsin-Madison) for the scanning of the Phantom.

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