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NON-STATIONARY NOISE ESTIMATION IN ACCELERATED PARALLEL MRI DATA

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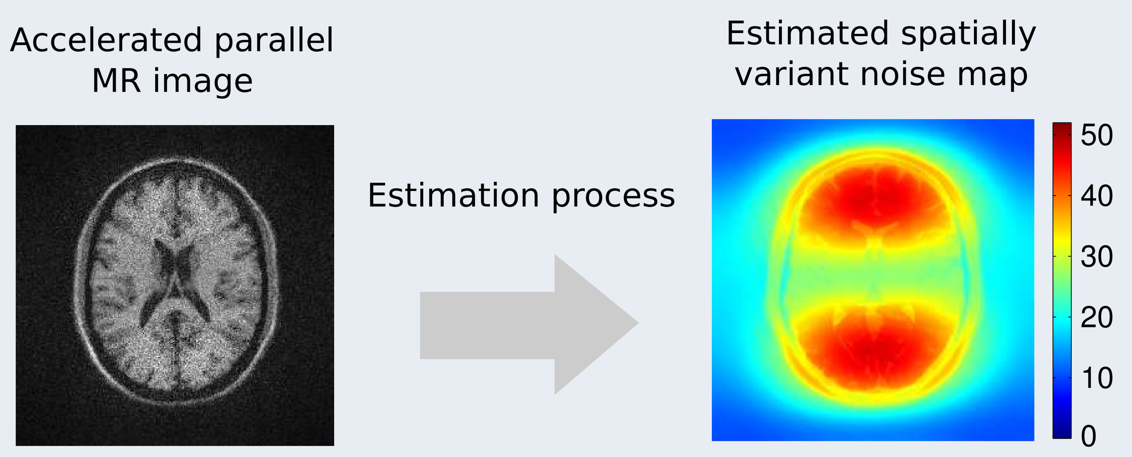
Abstract

The aim of this study is to retrieve spatially variant noise patterns from accelerated parallel MRI data using only a single image. Variance-stabilizing transformations (VSTs) for noncentral Chi (nc-χ) data are derived: (1) an analytic model and (2) a numerical model to improve the performance for low signal-to-noise ratios (SNRs). The VSTs generate Gaussian-like distributed variates from nc-χ data. The noise patterns are estimated then using Gaussian homomorphic filter.



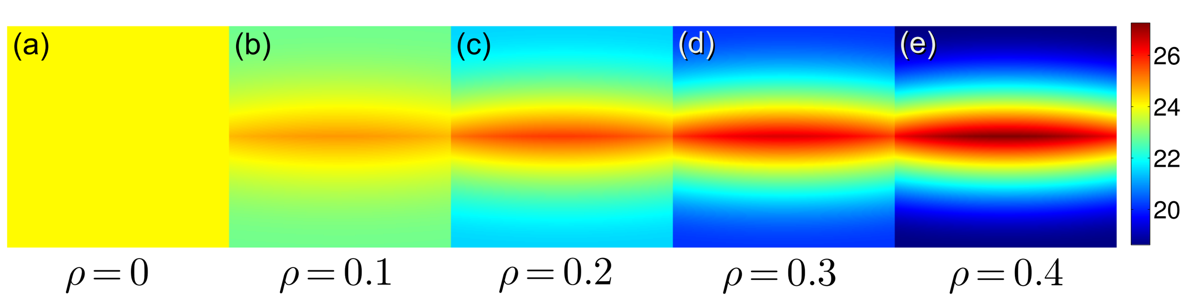
Non-stationary Rician noise

For Cartesian SENSE and Cartesian GRAPPA+SMF, the magnitude signal $M = M(\mathbf{x})$ follows a non-stationary Rician distribution:

$$p(M|A,\sigma) = \frac{M}{\sigma^2}\exp\left(-\frac{M^2+A^2}{2\sigma^2}\right) I_0\left(\frac{AM}{\sigma^2}\right),\ M \geq 0,$$

where:

- $A = A(\mathbf{x})$ is the amplitude signal,
- $\sigma^2 = \sigma^2(\mathbf{x})$ is the underlying noise variance.



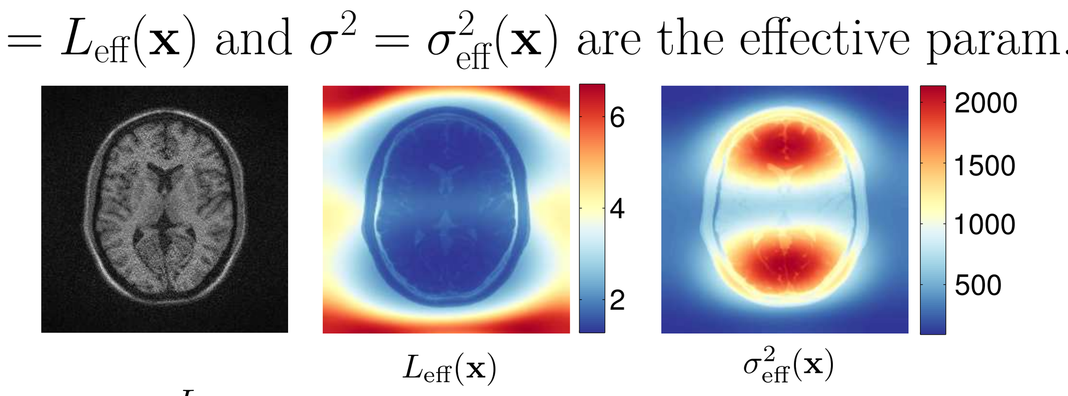
Non-stationary nc-χ noise

For Cartesian GRAPPA+SoS, the composite magnitude signal $M_L = M_L(\mathbf{x})$ can be modelled using a non-stationary nc-χ distribution with *effective parameters*:

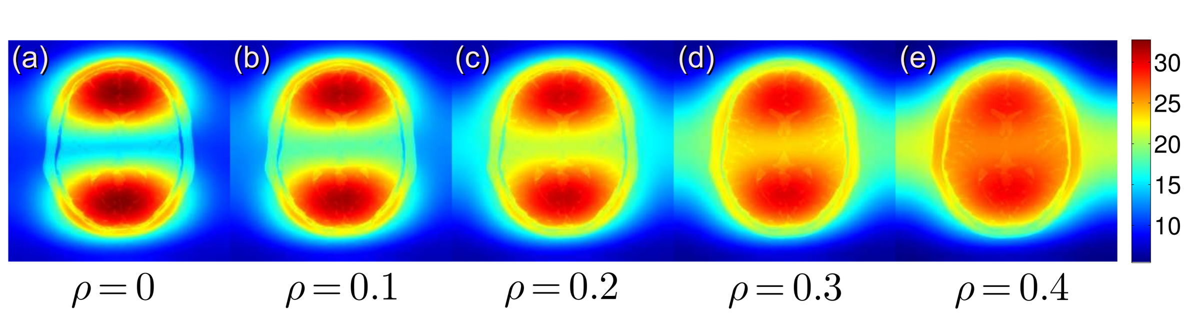
$$p(M_L|A_T,\sigma,L) = \frac{A_T^{1-L}}{\sigma^2} M_L^L \exp\left(-\frac{M_L^2+A_T^2}{2\sigma^2}\right) \times I_{L-1}\left(\frac{A_T M_L}{\sigma^2}\right),\ M_L \geq 0,$$

where:

- $L = L_{\mathrm{eff}}(\mathbf{x})$ and $\sigma^2 = \sigma^2_{\mathrm{eff}}(\mathbf{x})$ are the effective param.,



- $A_T^2(\mathbf{x}) = \sum_{l=1}^{L} |A_l^{\mathcal{R}}(\mathbf{x})|^2$,
- $I_k(\cdot)$ is the modified Bessel function of the first kind and k–th order.



The VST

The VST changes the signal-dependent noise in non-Gaussian data to a signal-independent one. We are interested therefore in a transformation $f_{\mathrm{stab}}\colon \mathbb{R} \mapsto \mathbb{R}$ that leads to a random variable with a constant variance, i.e., $\mathrm{Var}\{f_{\mathrm{stab}}(M_L|\sigma,L)\} = 1$.

The first-order Taylor expansion of f_{stab} is given by:

$$f_{\mathrm{stab}}(M_L|\sigma,L) = \int^{M_L} \frac{1}{\sqrt{\mathrm{Var}\{M_L|\widetilde{A_T},\sigma,L\}}}\, d\widetilde{A_T}.$$

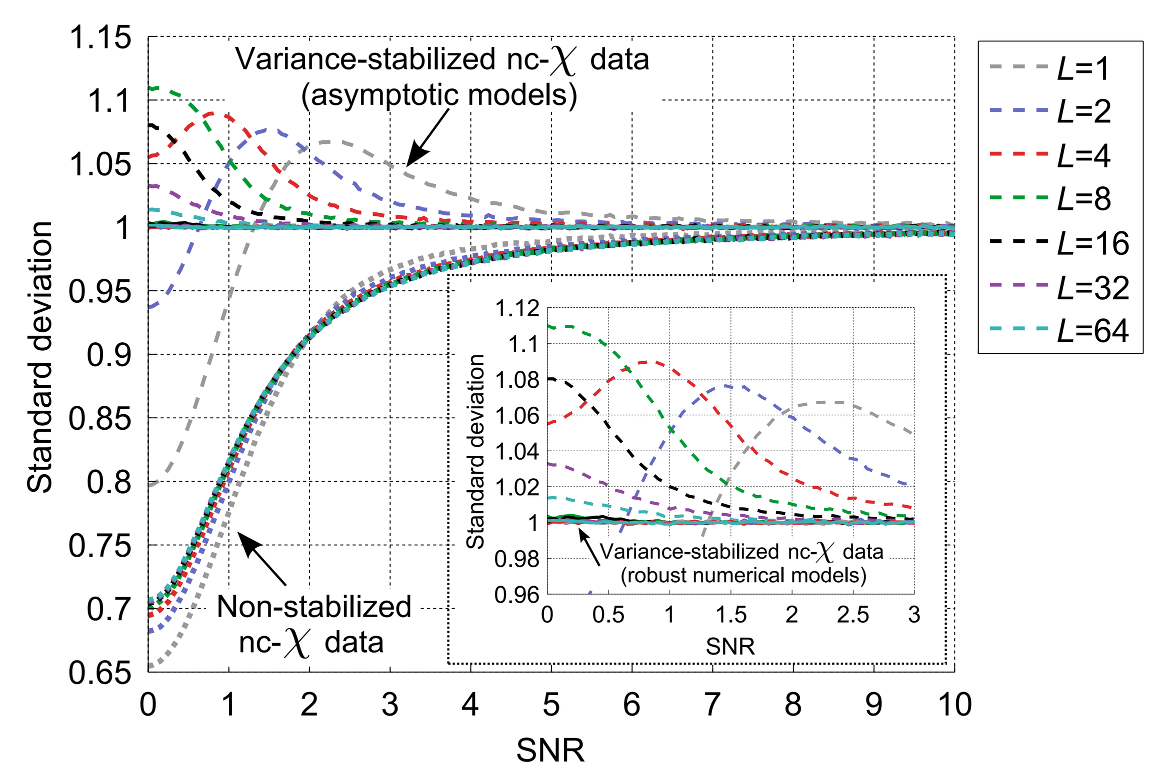
Problem! No closed-form for $\mathbb{E}\{M_L\}$ and $\mathrm{Var}\{M_L\}$.

- **Asymptotic model:**

Let $M_L^2 \sim$ nc-$\chi^2(A_T,\sigma,L)$. The VST is given by:

$$f_{\mathrm{stab}}(M_L^2|\sigma,L) = \frac{1}{\sigma}\sqrt{M_L^2 - L\sigma^2}.$$

This model is not optimal for low SNRs!



The VST (continuation)

- **Robust numerical model:**

A vector parameter $\mathbf{\Theta} = (\theta_1, \theta_2)$ is introduced:

$$f_{\mathrm{stab}}(M_L^2|\sigma,L,\mathbf{\Theta}) = \frac{1}{\sigma}\sqrt{\max\{\theta_1^2 M_L^2 - \theta_2 L\sigma^2,\ 0\}}.$$

The vector parameter $\mathbf{\Theta}$ must be tuned accordingly to the SNR of the signal, $\mathrm{SNR} = \frac{A_T}{\sqrt{L\sigma^2}}$:

$$\mathbf{\Theta}_{\mathrm{opt}} = \arg\min_{\mathbf{\Theta}} J\left(f_{\mathrm{stab}}(M_L^2|\sigma,L,\mathbf{\Theta})\right).$$

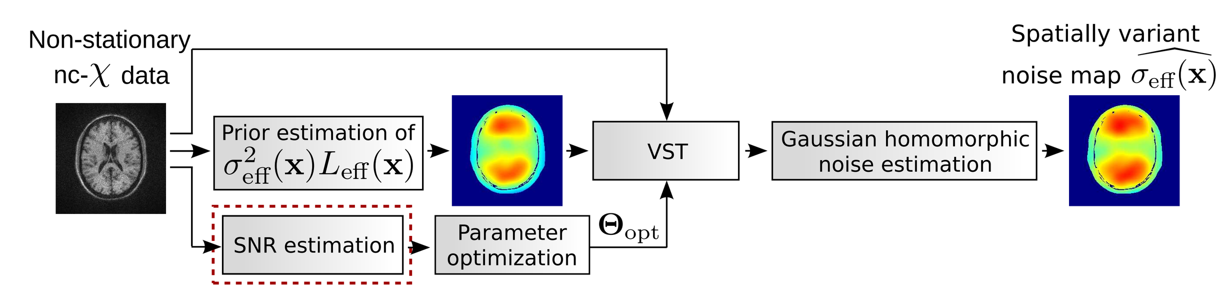
The cost function $J\colon \mathbb{R}^2 \mapsto \mathbb{R}$ is given then by:

$$\begin{aligned} J\left(f_{\mathrm{stab}}(\cdot|\sigma,L,\mathbf{\Theta})\right) &= \lambda_1 \cdot \varphi(1 - \mathrm{Var}\{f_{\mathrm{stab}}(\cdot|\sigma,L,\mathbf{\Theta})\}) \\ &+ \lambda_2 \cdot \varphi(\mathrm{Skew}\{f_{\mathrm{stab}}(\cdot|\sigma,L,\mathbf{\Theta})\}) \\ &+ \lambda_3 \cdot \varphi(\mathrm{ExKurt}\{f_{\mathrm{stab}}(\cdot|\sigma,L,\mathbf{\Theta})\}). \end{aligned}$$

The r–th raw moment for f_{stab}–trans. nc-χ^2 RV:

$$m_r = \int_0^{\infty} f_{\mathrm{stab}}^r(\widetilde{M_L^2}|\sigma,L,\mathbf{\Theta})\, \underbrace{p(\widetilde{M_L^2}|A_T,\sigma,L)}_{\text{PDF of nc-}\chi^2\text{ RV}}\, d\widetilde{M_L^2}$$

Spatially variant noise estimation



Gaussian homomorphic filter:

$$\widehat{\sigma_{\mathrm{eff}}(\mathbf{x})} = \sqrt{2}\exp\left(\mathrm{LPF}_{\sigma_f}\left\{\log\left|\widetilde{I}_{\mathrm{C}}(\mathbf{x})\right|\right\} + \frac{\gamma}{2}\right).$$

Noise estimation results

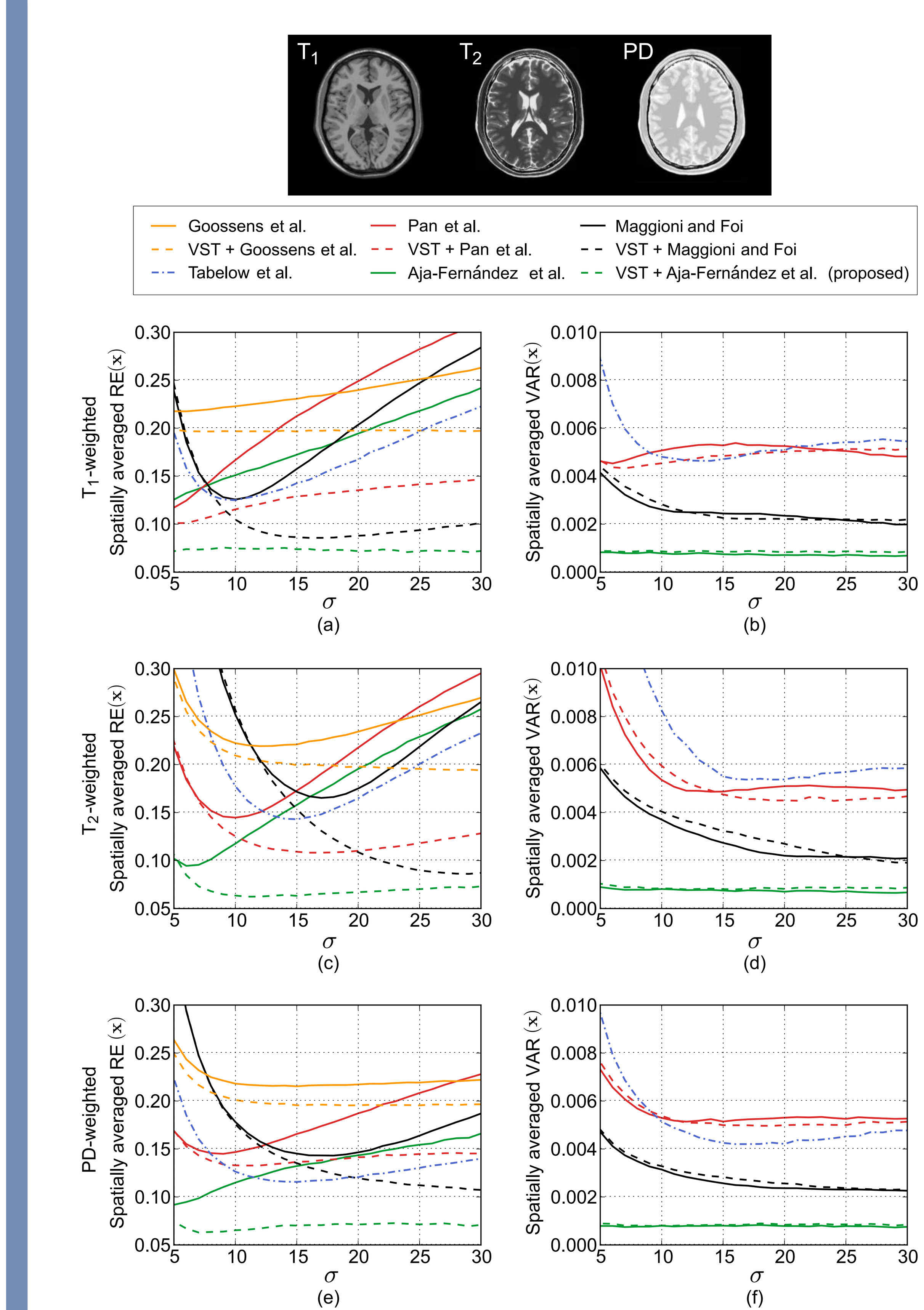


Figure 1: Noise estimators for synthetic GRAPPA MRI + SoS

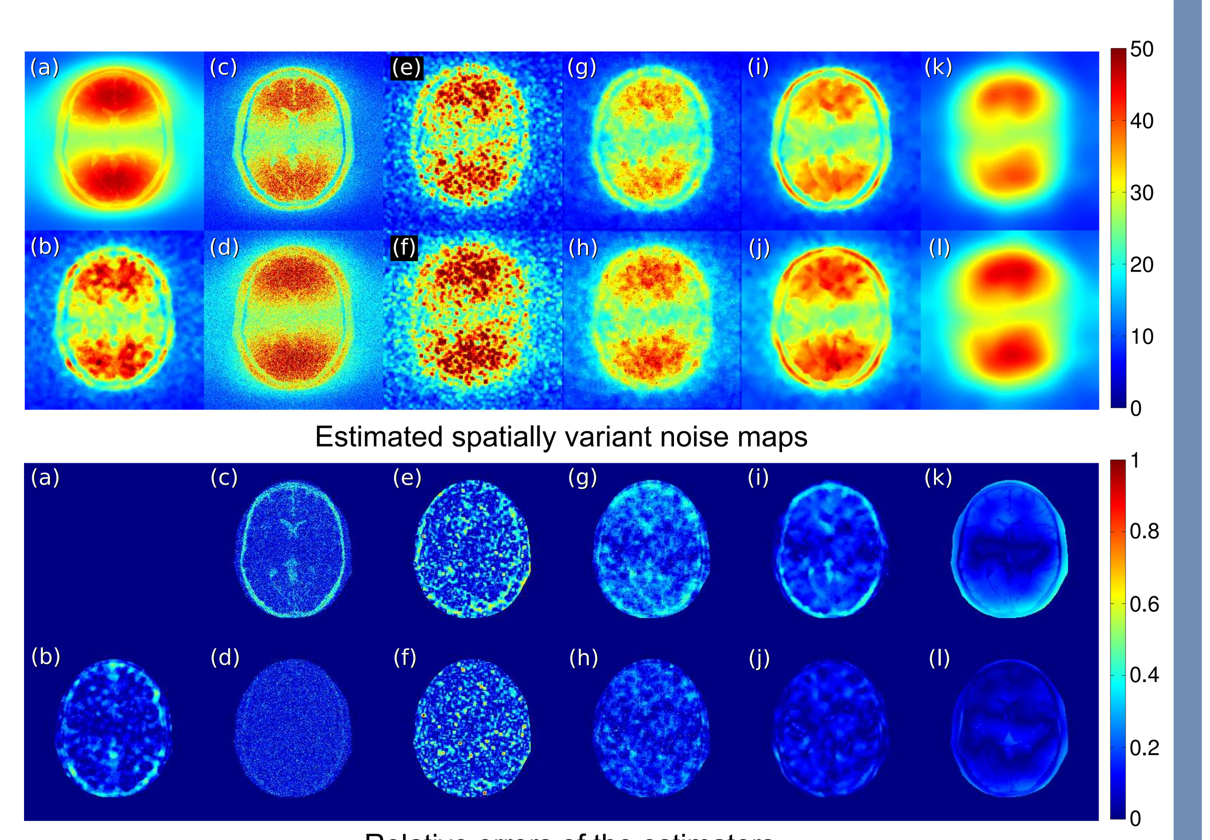


Figure 2: (a) *Ground truth*; (b) Tabelow; (c) std. dev. along samples; (d) VST+std. dev. along samples; (e) Goossens; (f) VST+Goossens, (g) Pan; (h) VST+Pan, (i) Maggioni, (j) VST+Maggioni, (k) Aja-Fernández, (l) **proposed**.

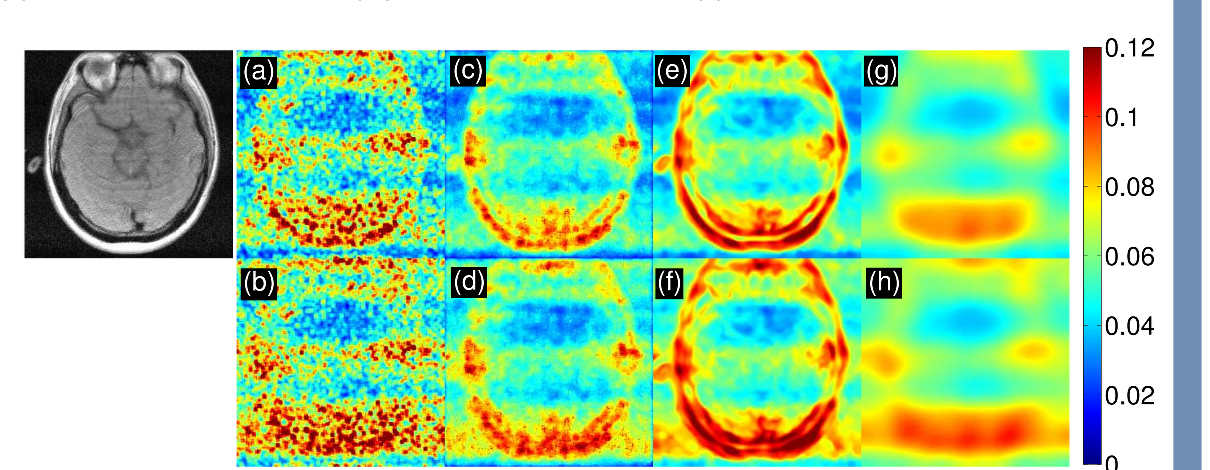


Figure 3: (a) Goossens; (b) VST+Goossens; (c) Pan; (d) VST+Pan, (e) Maggioni & Foi, (f) VST+Maggioni & Foi, (g) Aja-Fernández, (h) **proposed**.